Reply by Author to W. L. Oberkampf

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I WOULD like to thank Dr. Oberkampf for his interest in my Note.[1] However, it seems that his comments are, at least in part, a result of some misunderstanding.

First, the axial distance x is measured from the point of detachment of the vortex, and not the leading tip as in the Comment.[2] This was stated twice in my Note, but seems to have escaped Dr. Oberkampf's attention. This results in a shift of the "theory" line in Fig. 1 of the Comment toward the experimental data.

Another point raised is that the asymptotic angle is obtained only for asymmetric vortex wakes. However, in Refs. 3, 4, and others, symmetric wakes are also shown to separate from the body. The reference to Thomson and Morrison's[5] work was a) for the definition of ξ, and b) to show that this approach holds also for some of the more complex cases mentioned in my Note.[1]

The choice of data for Fig. 1 of the Comment[2] is somewhat surprising. The analysis is for incompressible flow, so the squares are of no relevance. On the other hand, for incompressible flow, Mendenhall and Nielsen[6] have a large collection of data. Their Figs. 5b and 5c have composite curves from various experimental results for vortex location. This can be adapted by my[1] Eq. (9) by calculating r from the horizontal and vertical positions for one point along the axis,

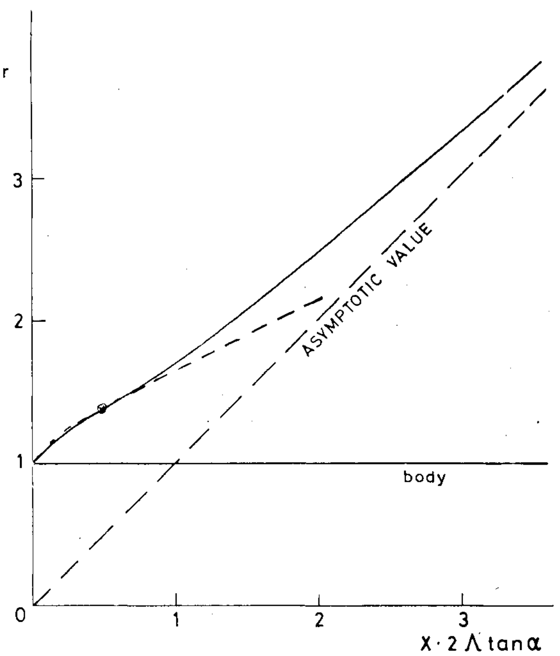

Fig. 1 Nondimensional distance of vortex from body centerline vs weighted nondimensional distance along centerline. Full line is theoretical prediction from Ref. 1 while broken line is composite experimental curve from Ref. 6. Circle indicates point used for calibration of the abscissa. Experimental data are available only for the range given.

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and calling it r. For example, taking the point Ref. 6 calls

$$\bar{x} = \frac{x - x_s}{r_n} \sin\alpha = 2$$

one obtains $r/a = 1.37$. From Eq. (9), this value of r/a leads to $x = 0.46$, i.e., a factor of $x = 0.23\,\bar{x}$ is required, where x is the coordinate in Fig. 1, and $\bar{x}$ is the axial coordinate in Ref. 6. This step is required for calibration of the abscissa as vortex separation points are not clearly defined.

One then obtains the broken line in Fig. 1, which shows agreement to within 20% for the entire range of available data. Reference 6 includes compilation of data from a wide range of sources, so that the compression of the axial coordinate required ($x/\bar{x} = 0.23$) will probably be rather accurate for most cases.

Finally, it should be recalled that the theory presented in Ref. 1 is extremely simplified and thus cannot, and should not be expected to, give very accurate predictions for vortex shedding. For example, Reynolds number effects on shedding position and vortex strength appear only indirectly. This method should be seen ony as an initial rapid estimation technique to help in the first iteration of more accurate numerical calculations of the flow around slender bodies at angles of attack, as mentioned previously.[1]

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Comment on "Finite Elements for Initial Value Problems in Dynamics"

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IN a recent paper,[1] Simkins made an attempt to formulate time finite elements based on Hamilton's "Law of Varying Action." It appears that, due to several misconceptions, the direct formulation of Ref. 1 "fails to yield a convergent sequence of solution." Consequently some mathematical manipulations have been introduced to achieve a solution. Thus, it is pertinent to make some comment referring to our

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earlier studies,[2-4] which, in our opinion, are free of such misconceptions and place the time element discretization of Hamilton's Law on a sound theoretical foundation. This is the aim of the following discussion.

Hamilton's Law,[2,5] in the absence of nonconservative forces, states that

$$\delta\int_{t_0}^{t_f}(T-V)\,dt-\sum_i\frac{\partial T}{\partial\dot{q}_i}\delta q_i\Big|_{t_0}^{t_f}=0 \qquad (1)$$

and serves as the basis for the present discussion. In Eq. (1), T and V are the kinetic and the potential energy of the system, respectively, and $q_i\,(i=1,...,n)$ are the generalized coordinates. In accordance with the finite element technique, the integral expression of Eq. (1) can be discretized and expressed as a sum over time finite elements.[3,4] By prescribing the same interpolation procedure as in Refs. 3 and 1, the values of q_i, $\dot{q}_i$ may be derived at any instant t at the element in terms of their values at the endpoints of this particular element. For the case of a single particle ($i=1$) having one degree-of-freedom, $u(t)$, this interpolation procedure and summation of the contributions of all of the elements yield the following expression for the integral and the summation term in Eq. (1).

$$H=U^T[(K-B)U-F] \qquad (2)$$

where K and B are $(2n+2)\times(2n+2)$ matrices obtained from $(T-V)$ and the second term, respectively, and $U^T=[u_0,\dot{u}_0,u_1,\dot{u}_1,...,u_n,\dot{u}_n]$.

At this stage it is important to repeat the fundamental axiomatic assumption given in Refs. 2 and 3: The physical solution of a given dynamic problem exists and is unique. The word "physical" means here that, although for some cases there may be more than one mathematical solution, for a real dynamic system, only one of them can be realized physically. This solution clearly depends on the system initial values. Once we know the existing solution, the state variables are known at any time, including the initial and the final state variables. In other words, there is a one to one relationship between the initial and the final values of the system. The essence of the above discussion is not changed if, in the sequel, one of these values is assumed to be known rather than being known. Based on the above assumption the following technique can be developed.[2-4] We assume that any possible combination of the initial and final coordinates and velocities are known; hence, their variations are equal to zero. Their number is taken to be equal to the number of the original initial conditions to make the formulation well defined. The reader will notice that q_i and $\dot{q}_i$, although assumed known, are not constrained at t_0 and t_f. Only some of their variations are imposed to be zero. Since these particular variations are no longer arbitrary, the equations they multiply equal some undefined constants, which can be different from zero. After introducing the correct initial values one can calculate these constants or eliminate them. For a one-degree-of-freedom dynamic system having two initial and two final state variables $(u,\dot{u})$, there exist six such possible combinations: $(\delta u_0\ \delta\dot{u}_0;\ \delta u_0\ \delta u_f;\ \delta\dot{u}_0\ \delta\dot{u}_f;\delta u_f\ \delta\dot{u}_f;\ \delta u_0\ \delta\dot{u}_f;\ \delta\dot{u}_0\ \delta u_f)$. A full discussion and exposition of all the possibilities require a full-length paper. The reader interested in details is referred to Refs. 2 and 4. Here we proceed with the possibility that $\delta u_0=\delta u_f=0^3$, which is the appropriate case for the paper in comment.

The first variation of Eq. (2) when $\delta u_0=\delta u_f=0$ yields the following system of equations,

$$(K-B)U-F=\psi \qquad (3)$$

where

$$\psi^T=[\psi_1,0,0,...,0,\psi_2,0]$$

where ψ_1 and ψ_2 are the two undefined constants. Equation (3) cannot be solved unless the initial values, e.g., u_0, $\dot{u}_0$, of the specific problem are introduced. In fact, they can be imposed on Eq. (3) in the same consistent manner as one imposes boundary conditions in standard finite element procedures.[3] It follows that the set (3) presents $2n+2$ equations for the $2n+2$ unknowns,

$$u_1,\dot{u}_1,u_2,\dot{u}_2,...,u_n,\dot{u}_n,\psi_1,\psi_2$$

The absence of ψ_2 in Ref. 1 is the reason for the "unknown" instability appearing there. Clearly, one can eliminate the two equations which correspond to ψ_1 and ψ_2. It follows that the restricted set (3) presents then $2n$ equations for the $2n$ unknowns. This elimination, for this and only this particular possibility $(\delta u_0=\delta u_f=0)$ under consideration, is equivalent to imposing

$$\frac{\partial T}{\partial\dot{u}}\delta u\Big|_{t_0}^{t_f}$$

to be zero, i.e., applying Hamilton's Principle, which requires the suppression of the two equations which correspond to u_0 and u_f from the system.

Thus, the same consistent and well defined set of equations for the initial value problem is achieved, either by employing Hamilton's Law or Hamilton's Principle, ruling out the need for mathematical manipulations, contrary to the formulation in Ref. 1. Following these procedures,[2-4] convergent solutions were obtained for the problem of the free oscillator considered by Simkins,[1] as well as for more complicated problems.

References

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Reply by Author to Riff, Weller, and Baruch

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THE lead paragraph of the Comment is seriously lacking in specificity, i.e., if the authors think I have "misconceptions," let them state what they are so that I may have a fair chance at rebuttal. Further, it should not be overlooked that my "attempt" to formulate a time-finite element formulation based on Hamilton's Law of Varying Action was a *successful* attempt in every way. The "mathematical manipulations" to which the authors have

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